

On the Problem of Radiation Friction Beyond 4 and 6 Dimensions

A. Mironov^a

Lebedev Physics Institute and ITEP, Moscow, Russia

A. Morozov^b

ITEP, Moscow, Russia

We count the number of independent structures which can arise in expressions for radiation friction force in different even space-time dimensions and demonstrate that their number is too big at $d \geq 8$ to allow determination of this force from the transversality condition alone, as was done by B.Kosyakov in $6d$. This implies that in general one can not bypass a tedious calculation involving explicit regularization and evaluation of emerging counterterms. However, simple Kosyakov's method works nicely in any dimension for the special case of circular motion with constant angular velocity.

1 Introduction

If radiation carries away the energy-momentum from a point like source at the rate W_μ , then the radiation friction force F_μ should appear at the r.h.s. of source's equation of motion,

$$m\dot{u}_\mu = f_\mu \longrightarrow m\dot{u}_\mu = f_\mu + F_\mu \quad (1.1)$$

where $u_\mu = \gamma(1, \vec{v})$, $\gamma = (1 - \vec{v}^2)^{-1/2}$ is relativistic velocity of the source, dot denotes derivative w.r.t. the particle self-time τ and f_μ is the relativistic force, which causes source's acceleration.

Energy-momentum conservation implies that the work of the radiation friction force should compensate the energy-momentum outflow

$$F_\mu = W_\mu + \dot{\xi}_\mu, \quad (1.2)$$

where the last term at the r.h.s. takes into account the change of the energy-momentum of electromagnetic field in the "near domain" around the source, which is not carried away to infinity, and is a total τ -derivative of an expression ξ_μ , made from velocity u and its τ -derivatives.

Since

$$u^2 = 1 \quad (1.3)$$

^aE-mail: mironov@itep.ru; mironov@lpi.ru

^bE-mail: morozov@itep.ru

the l.h.s. of eq.(1.1) is orthogonal to u ,

$$u\dot{u} = u^\mu \dot{u}_\mu = 0, \quad (1.4)$$

and so are relativistic forces at the r.h.s., $u^\mu f_\mu = 0$ and

$$u^\mu F_\mu = 0 \quad (1.5)$$

Evaluation of the radiation friction is a tricky task, far more sophisticated than that of W_μ , because it requires separation between the fields in the "near" and "far" (wave) domains and also involves discussion of interaction between the charge and its own field, related to celebrated problems like electromagnetic mass and Poincare tension. Still the force itself is a well defined – and even experimentally measurable – quantity, and one can be interested in knowing the answer for it irrespective of the details of the deep theory. Thus it is natural to search for short-cut ways to calculate radiation friction.

Note that our discussion of the radiation friction is formally applicable equally well for radiation of any spin s . However, for $s > 1$, the energy-momentum tensor for the point-like source is not conserved. This usually means that one cannot neglect contributions to radiation from tensions of the forces that cause acceleration of the source. Ultimately, it leads to additional contributions into the radiation friction.

2 Kosyakov's trick

The simplest and the most elegant option [1] is to take the well-known expression for W_μ [2] and to *construct* an expression for $\xi_\mu(u, \dot{u}, \dots)$, which satisfies orthogonality condition (1.5), i.e. adjust ξ_μ to satisfy

$$u^\mu \dot{\xi}_\mu + u^\mu W_\mu = 0 \quad (2.6)$$

Unfortunately, as explained below in this section, this trick, while effective in 4 and even in 6 dimensions, appears non-applicable in general, for $d \geq 8$.

Let $u_{kl} = \partial_\tau^k u^\mu \partial_\tau^l u_\mu$ denote scalar bilinears in τ -derivatives of u . Because of (1.3) they are not all independent: u_{0m} can be expressed through $u_{k,m-k}$ with $1 \leq k \leq l-1$:

$$\begin{aligned} u_{01} &= 0, \quad \text{see (1.4),} \\ u_{02} &= -u_{11}, \\ u_{03} &= -3u_{12}, \\ u_{04} &= -3u_{22} - 4u_{13}, \\ u_{05} &= -10u_{23} - 5u_{14}, \\ u_{06} &= -10u_{33} - 15u_{24} - 6u_{15}, \end{aligned} \quad (2.7)$$

...

$$u_{0m} = - \sum_{k=1}^{\left[\frac{m-1}{2}\right]} C_m^k u_{k,m-k} - \frac{1}{2} C_m^{m/2} u_{m/2,m/2}$$

The last term in the last formula is present only for even m , square brackets in the upper summation limit in the previous term denote integer part of $\frac{m-1}{2}$. Note in passing that the total number of items in the relation for u_{0m} is 2^{m-1} ; this is because they all arise as multiple derivatives of (1.3).

Expression for the radiated energy-momentum in d space-time dimensions (d even) looks as follows [3]:

$$W_\mu^{(d)} = \sum_{m=0}^{d-4} w_m^{(d)} \partial_\tau^m u_\mu \quad (2.8)$$

where $w_m^{(d)}$ is a polynomial in u_{kl} , a linear combination of N_{d-2-m} monomials $u_{k_1 l_1} \dots u_{k_r l_r}$ with any r (actually, $r \leq \frac{d-2-m}{2}$) and parameters k_i, l_i , $i = 1 \dots r$ constrained by the conditions

$$1 \leq k_i \leq l_i \quad (2.9)$$

and

$$\sum_{i=1}^r (k_i + l_i) = d - 2 - m. \quad (2.10)$$

On dimensional grounds also a term with $\partial_\tau^{d-2} u_\mu$ is allowed, but actually it does not contribute to $W_\mu^{(d)}$: the coefficient

$$w_{d-2}^{(d)} = 0 \quad (2.11)$$

There is no such a restriction in the case of $\xi_\mu^{(d)}$, instead it has one dimension less, and

$$\xi_\mu^{(d)} = \sum_{m=0}^{d-3} \kappa_m^{(d)} \partial_\tau^m u_\mu \quad (2.12)$$

with polynomials κ are similar to w , only this time $\sum_{i=1}^r (k_i + l_i) = d - 3 - m$. It follows that the scalars $u^\mu W_\mu$ and $u^\mu \xi_\mu$ are polynomials in u_{kl} with the total number $\sum(k + l) = d - 2$.

In order to check applicability of Kosyakov's trick we need to compare the two numbers:

$$\tilde{N}_{d-2} = \sum_{m=0}^{d-3} N_{d-3-m} \quad (2.13)$$

and N_{d-2} : the first one counts the total number of monomials, that *can* contribute to $\xi_\mu^{(d)}$, and the second one counts the number of monomials that can appear in $u^\mu W_\mu$ and which ξ_μ is supposed to compensate for according to (2.6). The trick works if $\tilde{N}_{d-2} = N_{d-2}$: then the condition (2.6) allows one to *unambiguously* extract all the coefficients in (2.12) from (2.6) – and this is indeed the case for $d = 4$ and $d = 6$. Unfortunately, for higher dimensions the matching breaks down: $\tilde{N}_{d-2} > N_{d-2}$ for $d \geq 8$, and ξ_μ is only constrained by (2.6), some $\tilde{N}_{d-2} - N_{d-2}$ coefficients in $\xi_\mu^{(d)}$ remain undefined, and other methods should be used in order to fix them unambiguously.

The numbers N_k are close relatives of the numbers n_k which count natural partitions of k ,

$$1 + \sum_{k=1}^{\infty} n_k q^k = \prod_{m=1}^{\infty} \frac{1}{1 - q^m}, \quad (2.14)$$

however, n_k would take only (2.10) into account, while in the case of N_k an additional constraint (2.9)

is imposed and members of every partition should be grouped in pairs:

1	$n_1 = 1 \quad N_1 = 0$
$2 = \underline{1+1}$	$n_2 = 2 \quad N_2 = 1$
$3 = 1 + 1 + 1 = \underline{1+2}$	$n_3 = 3 \quad N_3 = 1$
$4 = \underline{1+1+1+1} = 1 + 1 + 2 = \underline{1+3} = \underline{2+2}$	$n_4 = 5 \quad N_4 = 3$
$5 = 1 + 1 + 1 + 1 + 1 = \underline{1+1+1+2} = 1 + 1 + 3 = \underline{1+4} = 1 + 2 + 2 = \underline{2+3}$	$n_5 = 7 \quad N_5 = 3$
$6 = \underline{1+1+1+1+1+1} = 1 + 1 + 1 + 1 + 2 = \underline{1+1+1+3} = 1 + 1 + 4 =$ $= \underline{1+5} = \underbrace{1+1+2+2}_2 = 1 + 2 + 3 = 2 + 2 + 2 = \underline{2+4} = \underline{3+3}$	$n_6 = 11 \quad N_6 = 7$
...	

Underlined and under-braced are partitions, contributing to N_k , under-braced are partitions which contribute *several* times to N_k .

Generating function of numbers N_k is given by

$$\mathcal{N}(q) = \sum_{k=0}^{\infty} N_k q^k = \sum_{r=0}^{\infty} \mathcal{N}_r(q) \quad (2.15)$$

where $\mathcal{N}_r(q)$ counts the numbers of polynomials of degree r in u_{kl} . We have:

$$\mathcal{N}_0(q) = 1,$$

$$\begin{aligned} \mathcal{N}_1(q) &= q(q + q^2 + q^3 + \dots) + q^2(q^2 + q^3 + q^4 + \dots) + q^3(q^3 + q^4 + \dots) + \dots = \frac{q^2}{(1-q)(1-q^2)} = \\ &= \sum_{k=1}^{\infty} n_{1k} q^k = q^2 + q^3 + 2q^4 + 2q^5 + 3q^6 + 3q^7 + 4q^8 + 4q^9 + \dots = \sum_{k=1}^{\infty} k \left(q^{2k} + q^{2k+1} \right), \end{aligned} \quad (2.16)$$

$$\begin{aligned} \mathcal{N}_2(q) &= \frac{1}{2} \left(\mathcal{N}_1^2(q) + \mathcal{N}_1(q^2) \right) = \frac{q^4(1+q^3)}{(1-q)(1-q^2)^2(1-q^4)} = \\ &= q^4 + q^5 + 3q^6 + 4q^7 + 8q^8 + 10q^9 + 16q^{10} + \dots, \\ \mathcal{N}_3(q) &= \frac{1}{6} \left(\mathcal{N}_1^3(q) + 3\mathcal{N}_1(q^2)\mathcal{N}_1(q) + 2\mathcal{N}_1(q^3) \right) \\ &= q^6 + q^7 + 3q^8 + \dots, \end{aligned}$$

$$\mathcal{N}_4(q) = \frac{1}{24} \left(\mathcal{N}_1^4(q) + 6\mathcal{N}_1(q^2)\mathcal{N}_1^2(q) + 3\mathcal{N}_1^2(q^2) + 8\mathcal{N}_1(q^3)\mathcal{N}_1(q) + 6\mathcal{N}_1(q^4) \right)$$

and so on. Collecting all terms, we obtain:

$$\begin{aligned} \mathcal{N}(q) &= \sum_{r=0}^{\infty} \mathcal{N}_r(q) = \exp \left(\mathcal{N}_1(q) + \frac{1}{2}\mathcal{N}_1(q^2) + \frac{1}{3}\mathcal{N}_1(q^3) + \dots \right) = \exp \left(\sum_{k=1}^{\infty} \frac{1}{k} \mathcal{N}_1(q^k) \right) = \\ &\stackrel{(2.16)}{=} \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^{n_{1k}}} = \prod_{k=1}^{\infty} \frac{1}{(1-q^{2k})^k (1-q^{2k+1})^k} \end{aligned} \quad (2.17)$$

The product $\prod_{k=1}^{\infty} (1-q^{2k})^{-k}$ is familiar from the theory of $3d$ partitions (McMahon), it plays the same role as $\prod_{k=1}^{\infty} (1-q^k)^{-1}$ from (2.14) for ordinary partitions. According to (2.13) the generating function for the numbers \tilde{N}_q is equal to

$$\tilde{\mathcal{N}}(q) = \frac{q\mathcal{N}(q)}{1-q} \quad (2.18)$$

The first few numbers N_k are given in the following table, where also the relevant monomials are explicitly listed:

k	N_k	monomials
0	1	$1 = u^2 = u_{00}$
1	0	$0 = (u\dot{u}) = u_{01}$
2	1	$u_{11} = \dot{u}^2$
3	1	u_{12}
4	3	u_{11}^2, u_{13}, u_{22}
5	3	$u_{11}u_{12}, u_{14}, u_{23}$
6	7	$u_{11}^3, u_{12}^2, u_{11}u_{13}, u_{11}u_{22}, u_{15}, u_{24}, u_{33}$
7	8	$u_{11}^2u_{12}, u_{11}u_{14}, u_{11}u_{23}, u_{12}u_{13}, u_{12}u_{22}, u_{16}, u_{25}, u_{34}$
8	16	$u_{11}^4, u_{11}^2u_{13}, u_{11}^2u_{22}, u_{12}^2u_{11}, u_{11}u_{15}, u_{11}u_{24}, u_{11}u_{33}, u_{12}u_{14}, u_{12}u_{23}, u_{13}^2, u_{13}u_{22}, u_{22}^2, u_{17}, u_{26}, u_{35}, u_{44}$

It is now easy to find \tilde{N}_{d-2} and compare them with N_{d-2} :

d		4		6		8		10
$k = d - 2$	1	2	3	4	5	6	7	8
N_k	0	1	1	3	3	7	8	16
\tilde{N}_k	1	1	2	3	6	9	16	24

For $d = 4$ we have the exact matching, $\tilde{N}_2 = N_2 = 1$, in more details [2, 3] (here s is the spin of radiation)

$$\begin{aligned}
W_\mu^{(4)} &= -\frac{4-12s}{3}\pi u_\mu u_{11}, \\
u^\mu W_\mu^{(4)} &= -\frac{4-12s}{3}\pi u_{11} \quad \text{-- only one term, because } N_2 = 1, \\
\xi_\mu^{(4)} &= -\frac{4-12s}{3}\pi \dot{u}_\mu \quad \text{-- only one term, because } \tilde{N}_2 = 1, \\
F_\mu^{(4)} &= W_\mu^{(4)} + \dot{\xi}_\mu^{(4)} = -\frac{4-12s}{3}\pi(\ddot{u}_\mu + u_\mu u_{11})
\end{aligned} \tag{2.19}$$

In the non-relativistic limit, the first term dominates and

$$\vec{F}^{(4)} \approx -\frac{4-12s}{3}\pi \ddot{\vec{v}}. \tag{2.20}$$

Similarly, for $d = 6$ we also have the exact matching, $\tilde{N}_4 = N_4 = 3$, in more details [1, 3]

$$\begin{aligned}
W_\mu^{(6)} &= u_\mu(\alpha u_{22} + \beta u_{11}^2) + \gamma \dot{u}_\mu u_{12} + \delta \ddot{u}_\mu u_{11}, \\
u^\mu W_\mu^{(6)} &= \alpha u_{22} + (\beta - \delta)u_{11}^2 \quad \text{-- three terms contribute, because } N_3 = 1, \\
\xi_\mu^{(6)} &= -4\alpha u_\mu u_{12} + (\beta - \delta)\dot{u}_\mu u_{11} - \alpha \ddot{u}_\mu \quad \text{-- three terms contribute, because } \tilde{N}_4 = 1, \\
F_\mu^{(6)} &= W_\mu^{(6)} + \dot{\xi}_\mu^{(6)} = -\alpha \ddot{u}_\mu + \beta \ddot{u}_\mu u_{11} + (\gamma - 4\alpha + 2\beta + 2\delta)\dot{u}_\mu u_{12} + u_\mu(\beta u_{11}^2 - 3\alpha u_{22} - 4\alpha u_{13})
\end{aligned} \tag{2.21}$$

The coefficients here are equal to

$$\begin{aligned}
\alpha &= \frac{8\pi^2}{15}(1-5s), \quad \beta = \pi^2 \left[\frac{19}{3} - (2s-3)^2 \right] \\
\gamma &= \frac{16\pi^2}{35}(2-7s), \quad \delta = \frac{16\pi^2}{105}(7s-4)
\end{aligned} \tag{2.22}$$

In the non-relativistic limit, the first term dominates (this is the case in all dimensions!), and

$$\vec{F}^{(6)} \approx -\alpha \ddot{\vec{v}} = \frac{8\pi^2}{15}(5s-1)\ddot{\vec{v}} \tag{2.23}$$

In fact, in the expressions above the sign of W_μ was chosen so that it describes correctly all s but $s = 0$ (scalar). In the latter case, one should reverse the sign. This is because in all but the scalar cases only the spatial components of all non-zero spin fields have any physical meaning (e.g. survive in physical gauges), thus giving rise to the overall minus sign of the kinetic part of the energy-momentum tensor as compared with the scalar case. This means that one should also reverse the sign of ξ_μ and \vec{F} . Let us stress again that the results for only $s = 0, 1$ (scalar and electromagnetic radiations) have practical applicability (see the Introduction).

Unfortunately, already for $d = 8$ the matching fails, $\tilde{N}_6 = 9 > N_6 = 7$. Mismatch does not allow one to define the coefficients in front of $2 = \tilde{N}_6 - N_6$ structures, which can potentially contribute to $\xi_\mu^{(8)}$, but remain unconstrained by (2.6). These two structures are:

$$\zeta_\mu^{(8,1)} = 3u_\mu u_{11} u_{12} + \dot{u}_\mu (u_{13} + u_{22}) + 2\ddot{u}_\mu u_{12} \quad (2.24)$$

and

$$\zeta_\mu^{(8,2)} = 3u_\mu u_{11} u_{12} - \dot{u}_\mu u_{13} + \ddot{u}_\mu u_{11} \quad (2.25)$$

– it is easy to check that the τ -derivative of any of the two is orthogonal to u^μ :

$$u^\mu \dot{\zeta}_\mu \equiv 0. \quad (2.26)$$

3 Circular motion

The above calculus becomes somewhat different in the special when the source moves along a circular orbit with constant value of velocity, namely when \vec{v}^2 and thus $u_0 = \gamma$ do not change with time. The spatial vector $\vec{u} = \gamma \vec{v}$ changes direction, but the acceleration is orthogonal to the velocity, $\vec{v} \cdot \dot{\vec{v}} = 0$, and, as a corollary, all

$$u_{kl} = 0 \quad \text{if} \quad k + l \quad \text{is odd} \quad (3.27)$$

Taking time-derivative of these relations, we obtain

$$u_{k,l+1} + u_{k+1,l} = 0 \quad \text{if} \quad k + l + 1 \quad \text{is even}, \quad (3.28)$$

for example, $u_{13} + u_{22} = \dot{u}_{12} = 0$. In addition,

$$\partial_\tau^k u_\mu \partial_\tau^l u_\nu = \partial_\tau^k u_\mu \partial_\tau^l u_\nu \quad \text{for} \quad k < l \quad \text{and even difference} \quad l - k, \quad (3.29)$$

for example, $\dot{u}_\mu \ddot{u}_\nu = \dot{u}_\nu \ddot{u}_\mu$. An immediate corollary of relations (3.27)-(3.29) is that both $\zeta_\mu^{(8,1)} = \zeta_\mu^{(8,2)} = 0$. This means that *for circular motion* there is no uncertainty and Kosyakov's trick is sufficient to determine the radiation friction unambiguously, at least, for $d = 8$.

In fact, this is true also for $d = 10$ and, moreover, for all even dimensions d . In general, we have for circular motion with constant velocity:

d	$k = d - 2$	monomials	N_k^{circ}	\tilde{N}_k^{circ}
2	0	$1 = u^2 = u_{00}$	1	
	1	$0 = (u\dot{u}) = u_{01}$	0	
4	2	$u_{11} = \dot{u}^2$	1	1
	3	u_{12}	0	
6	4	$u_{11}^2, u_{13} = -u_{22}$	2	2
	5	$u_{11}u_{12} = u_{14} = u_{23} = 0$	0	
8	6	$u_{11}^3, u_{11}u_{13} = -u_{11}u_{22}, u_{15} = -u_{24} = u_{33}; u_{12}^2 = 0,$	3	$4 - 1 = 3$
	7	$u_{11}^2u_{12} = u_{11}u_{14} = u_{11}u_{23} = u_{12}u_{13} = u_{12}u_{22} = u_{16} = u_{25} = u_{34} = 0$	0	
10	8	$u_{11}^4, u_{11}^2u_{13} = -u_{11}^2u_{22}, u_{11}u_{15} = -u_{11}u_{24} = u_{11}u_{33} [=] u_{13}^2 = -u_{13}u_{22} = u_{22}^2,$ $u_{17} = -u_{26} = u_{35} = -u_{44}; u_{12}^2u_{11} = 0, u_{12}u_{14} = u_{12}u_{23} = 0,$	$5 - 1$ $= 4$	$7 - 3$ $= 4$

Numbers N_k^{circ} differ from N_k in the previous tables, because of relations (3.27) and (3.28). Only *bilinear* combinations of the last relations (3.29) affect N_k^{circ} , and this happens for the first time for $k = 8$, i.e. $d = 10$. Then the square of (3.29) with $k, l = 1, 3$ implies an additional relation between monomials, which is denoted by [=] in the table and subtracts 1 in evaluating the number N_k^{circ} .

If (3.29) is not taken into account, then the numbers \tilde{N}_k^{circ} are defined by the same relation (2.13), only now N_k^{circ} enter the r.h.s. instead of N_k . However, (2.13) requires an additional modification,

which takes (3.29) into account, and this modification is *linear* in (3.29). For example, from $\tilde{N}_6^{circ} = 4$ one still needs to subtract 1, associated with potential, but vanishing due to (3.29) contribution $-\dot{u}_\mu u_{13} + \ddot{u}_\mu u_{11} = (-\dot{u}_\mu \ddot{u}_\nu + \dot{u}_\nu \ddot{u}_\mu) \dot{u}^\nu = 0$ to $\xi_\mu^{(8)}$. Similarly, from $\tilde{N}_8^{circ} = 7$ one needs to subtract 3, associated with three such structures $0 = (-\dot{u}_\mu \ddot{u}_\nu + \dot{u}_\nu \ddot{u}_\mu) \ddot{u}^\nu = -\dot{u}_\mu u_{33} + \ddot{u}_\mu u_{13}$, $0 = (-\dot{u}_\mu \ddot{u}_\nu + \dot{u}_\nu \ddot{u}_\mu) \dot{u}^\nu u_{11} = -\dot{u}_\mu u_{11} u_{13} + \ddot{u}_\mu u_{11}^2$ and $0 = (-\dot{u}_\mu \ddot{u}_\nu + \dot{u}_\nu \ddot{u}_\mu) \dot{u}^\nu = -\dot{u}_\mu u_{15} + \ddot{u}_\mu u_{11}$.

The table demonstrates that $\tilde{N}_k^{circ} = N_k^{circ}$ for all even d , at least, till $d = 10$, and this justifies the use of Kosyakov's trick for evaluation of the radiation friction *for the circular motion* for these dimensions.

The generating function

$$\mathcal{N}_1^{circ}(q) = \frac{q^2}{1-q^2} = (1-q)\mathcal{N}_1(q) \quad (3.30)$$

Thus, before subtractions,

$$\mathcal{N}^{circ}(q) = \exp\left(\sum_{k=1}^{\infty} \frac{1}{k} \mathcal{N}_1^{circ}(q)\right) = \prod_{k=1}^{\infty} \frac{1}{1-q^{2k}} \quad (3.31)$$

and, also before subtractions,

$$\tilde{\mathcal{N}}^{circ}(q) = \frac{q}{1-q} \mathcal{N}^{circ}(q) \quad (3.32)$$

4 Conclusion

Thus, we conclude that the elegant method, successfully used by B.Kosyakov to evaluate the radiation friction force in 4 and 6 space-time dimensions, can not be directly used in general situation in higher dimensions (though it still works nicely for the circular motion with constant angular velocity). Therefore, it seems unavoidable to make full-scale calculations with explicit regularization and counter-terms in the action, as it has been done in $6d$ in [4]. For first results in this direction beyond 6 dimensions, see [5]. Among interesting questions, appearing on this way, is the counterterms dependence on the choice of regularization (if all counterterms can be varied independently) and physical interpretation of emerging corrections to the naive action of relativistic particle [6].

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L.Landau and E.Lifshitz, *The Classical Theory of Fields (Course of Theoretical Physics, Volume 2)*

J.D.Jackson, *Classical Electrodynamics*, Wiley, New York, 1975

For additional peculiarities of generalization to gravitational radiation, see

I.Khriplovich, *General Relativity* (in Russian), Izhevsk, 2001

For a textbook, presenting a through discussion of all the details, also in the case of $d = 6$, but only for electromagnetic radiation, see

B.Kosyakov, *Introduction to the Classical Theory of Particles and Fields*, Springer, 2007

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For exhaustive set of formulas for any even d see [3]. This paper contains expression also for radiated fields of all spins s , but for $s > 1$ the radiation problem for a point-like source is not well-defined, so these formulas are not of direct applicability – contribution of radiation from other sources, standing behind the force f_μ in (1.1) should be also included.

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